Homework 1 Solutions

Ch. 3: 6, 8, 10, 12, 22, 24, 46

3.6

(a) $(3.5+8.0-5.0)\pm(0.1+0.2+0.4)=6.5\pm0.7$

(b) $(3.5\times8.0)\left(1\pm\left(\frac{0.1}{3.5}+\frac{0.2}{8.0}\right)\right)=28.0(1\pm0.054)=28.0\pm1.5$

(c) $\frac{8.0}{5.0}\left(1\pm\left(\frac{0.2}{8.0}+\frac{0.4}{5.0}\right)\right)=1.60(1\pm0.11)=1.60\pm0.18$

(d) $\frac{3.5\times8.0}{5.0}\left(1\pm\left(\frac{0.1}{3.5}+\frac{0.2}{8.0}+\frac{0.4}{5.0}\right)\right)=5.60(1\pm0.13)=5.6\pm0.7$

3.8

(a) Let n be a positive integer. Then, the $n+1$th term will contain a factor of n-n = 0 in the numerator, and will therefore be zero. All subsequent terms will also contain this factor, and will therefore also be zero. So, the series becomes a finite polynomial.

For n = 2: $(1+x)^2=1+nx+\frac{1}{2}n(n-1)x^2=1+2x+x^2$

For n = 3: $(1+x)^3=1+nx+\frac{1}{2}n(n-1)x^2+\frac{1}{6}n(n-1)(n-2)x^3=1+3x+3x^2+x^3$

(b) For n = -1, the series is:

$$(1+x)^{-1}=1+nx+\frac{1}{2}n(n-1)x^2+\frac{1}{6}n(n-1)(n-2)x^3+...=1-x+x^2-x^3+...=\sum_{m=0}^{\infty}(-x)^m$$

Comparing first-order approximation, $(1+x)^{-1}\approx1-x$, to exact value:

x = 0.5	1 - x = 0.5	1/(1 + x) = 2/3	Error = 25%
x = 0.1	1 - x = 0.9	1/(1 + x) = 10/11	Error = 1.0%
x = 0.01	1 - x = 0.99	1/(1 + x) = 100/101	Error = 0.010%

Clearly, the % error rapidly gets very small as x approaches zero.

3.10

(a) The thickness of one card is $\dfrac{0.590 \pm 0.005 in}{52} = (1.135 \pm 0.010) \times 10^{-2} in$

(b) The uncertainty is inversely proportional to the number of cards measured (since it is divided by the number of cards). We want an uncertainty of $2 \times 10^{-5} in$. That is 5 times less than in part (a), so we need at least 5 decks of cards.

3.12

Since % uncertainty for x is 0.1 / 4.0 = 2.5%, we have

$$x^2 = 4.0^2(1 \pm 2 \times 0.025) = 16.0(1 \pm 0.05) = 16.0 \pm 0.08 \qquad \text{5\% uncertainty}$$

$$x^3 = 4.0^3(1 \pm 3 \times 0.025) = 64.0(1 \pm 0.075) = 64 \pm 5 \qquad \text{7.5\% uncertainty}$$

3.22

(a) $$P = IV = (2.10 \pm 0.02 A)(1.02 \pm 0.01 V) = 2.10 \times 1.02 W \left(1 \pm \sqrt{\left(\frac{0.02}{2.10}\right)^2 + \left(\frac{0.01}{1.02}\right)^2}\right)$$

$$= 2.142 W(1 \pm 0.014) = 2.14 \pm 0.03 W$$

(b) $$R = V / I = 0.485(1 \pm 0.014)\Omega = 0.485 \pm 0.007\Omega$$

3.24

$$r = \frac{125}{32 \times (4\pi \times 10^{-7} N / A^2)^2 \times 72^2} \frac{(661 mm)^2 \times 45.0 V}{(91.4 mm)^2 (2.48 A)^2} \times$$

$$\times \left(1 \pm \sqrt{4\left(\frac{2}{661}\right)^2 + \left(\frac{0.2}{45.0}\right)^2 + 4\left(\frac{0.5}{91.4}\right)^2 + 4\left(\frac{0.04}{2.48}\right)^2}\right) = (1.83 \pm 0.06) \times 10^{11} C / kg$$

3.46

$$\frac{\partial q}{\partial x} = y + \frac{2x}{y} \approx 3.0 + \frac{2 \cdot 6.0}{3.0} = 7.0 \qquad \frac{\partial q}{\partial y} = x - \frac{x^2}{y^2} \approx 6.0 - \frac{36.0}{9.0} = 2.0$$

Thus, $q = 30.0 \pm \sqrt{(7.0 \cdot 0.1)^2 + (2.0 \cdot 0.1)^2} = 30.0 \pm 0.7$